Consistent Subtyping for All

Ningning Xie **Xuan Bi** Bruno C. d. S. Oliveira

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The University of Hong Kong

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Gradual Typing 101

- The key external feature of every gradual type system is the *unknown type* $\star$.

```
f (x : Int) = x + 2    -- static checking
h (g : ⋆) = g 1        -- dynamic checking
h f
```

- Central to gradual typing is type consistency $\sim$, which relaxes type equality: $\star \sim \mathsf{Int}$, $\star \to \mathsf{Int} \sim \mathsf{Int} \to \star, \ldots$
- Dynamic semantics is defined by type-directed translation to an internal language with runtime casts:

$$(\langle \star \hookrightarrow \star \to \star \rangle g)\ (\langle \mathsf{Int} \hookrightarrow \star \rangle 1)$$

Many Successes

Gradual typing has seen great popularity both in academia and industry. Over the years, there emerge many gradual type disciplines:

- Subtyping
- Parametric Polymorphism
- Type inference
- Security Typing
- Effects
- ...

Many Successes, But...

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☞ *As type systems get more complex, it becomes more difficult to adapt notions of gradual typing.* [Garcia et al., 2016]

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- State-of-art techniques are inadequate.

Why It Is interesting

- Haskell supports implicit higher-rank polymorphism, but some "safe" programs are rejected:

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foo :: ([Int], [Char])
foo = let f x = (x [1, 2], x ['a', 'b'])
      in f reverse  -- GHC rejects
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- If we had gradual typing...

```
let f (x : ⋆) = (x [1, 2], x ['a', 'b'])
in f reverse
```

- Moving to more precised version still type checks, but with more static safety guarantee:

```
let f (x : ∀a. [a] → [a]) = (x [1, 2], x ['a', 'b'])
in f reverse
```

Contributions

- A new specification of consistent subtyping that works for implicit higher-rank polymorphism
- An easy-to-follow recipe for turning subtyping into consistent subtyping
- A gradually typed calculus with implicit higher-rank polymorphism
    - Satisfies correctness criteria (formalized in Coq)
    - A sound and complete algorithm

What Is Consistent Subtyping

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- An algorithm for consistent subtyping in terms of masking $A|_B$

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- An algorithm for consistent subtyping in terms of masking $A|_B$

Definition (Consistent Subtyping à la Siek and Taha)

The following two are *equivalent*:

1. $A \lesssim B$ if and only if $A \sim C$ and $C <: B$ for some C.
2. $A \lesssim B$ if and only if $A <: C$ and $C \sim B$ for some C.

Design Principle

☞ *Gradual typing and subtyping are orthogonal and can be combined in a principled fashion.* *– Siek and Taha*

Challenge

- Polymorphic types induce a subtyping relation:
  $\forall a.\, a \to a <: \mathsf{Int} \to \mathsf{Int}$
- Design consistent subtyping that combines 1) consistency 2) subtyping 3) polymorphism.

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[1] Note that here we are mostly concerned with static semantics.

Problem with Existing Definition

Odersky-Läufer Type System

- The underlying static language is the well-established type system for higher-rank types. [Odersky and Läufer, 1996]

Types	A, B	$::=$	$\mathsf{Int} \mid a \mid A \to B \mid \forall a.\, A$
Monotypes	τ, σ	$::=$	$\mathsf{Int} \mid a \mid \tau \to \sigma$
Terms	e	$::=$	$x \mid \mathsf{n} \mid \lambda x : A.\, e \mid \lambda x.\, e \mid e_1\ e_2$
Contexts	Ψ	$::=$	$\bullet \mid \Psi, x : A \mid \Psi, a$

Subtyping

$\boxed{\Psi \vdash A <: B}$ *(Subtyping)*

$$\frac{a \in \Psi}{\Psi \vdash a <: a} \qquad \frac{}{\Psi \vdash \mathsf{Int} <: \mathsf{Int}} \qquad \frac{\Psi \vdash B_1 <: A_1 \qquad \Psi \vdash A_2 <: B_2}{\Psi \vdash A_1 \to A_2 <: B_1 \to B_2}$$

$$\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] <: B}{\Psi \vdash \forall a.\, A <: B} \qquad \frac{\Psi, a \vdash A <: B}{\Psi \vdash A <: \forall a.\, B}$$

Subtyping with Unknown Types

$\boxed{\Psi \vdash A <: B}$ *(Subtyping)*

$$\frac{a \in \Psi}{\Psi \vdash a <: a} \qquad \frac{}{\Psi \vdash \mathsf{Int} <: \mathsf{Int}} \qquad \frac{\Psi \vdash B_1 <: A_1 \qquad \Psi \vdash A_2 <: B_2}{\Psi \vdash A_1 \to A_2 <: B_1 \to B_2}$$

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$$\frac{}{\Psi \vdash \star <: \star}$$

Type Consistency

$\boxed{A \sim B}$ *(Type Consistency)*

$$\frac{}{A \sim A} \qquad \frac{}{A \sim \star} \qquad \frac{}{\star \sim A} \qquad \frac{A_1 \sim B_1 \qquad A_2 \sim B_2}{A_1 \to A_2 \sim B_1 \to B_2}$$

Type Consistency with Polymorphic Types

$\boxed{A \sim B}$ *(Type Consistency)*

$$\frac{}{A \sim A} \qquad \frac{}{A \sim \star} \qquad \frac{}{\star \sim A} \qquad \frac{A_1 \sim B_1 \qquad A_2 \sim B_2}{A_1 \to A_2 \sim B_1 \to B_2}$$

$$\frac{A \sim B}{\forall a.\, A \sim \forall a.\, B}$$

Type Consistency with Polymorphic Types

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$$\frac{A \sim B}{\forall a.\, A \sim \forall a.\, B}$$

☞ *The simplicity comes from the orthogonality between consistency and subtyping!*

Bad News

Definition (Consistent Subtyping à la Siek and Taha)

The following two are equivalent:

1. $A \lesssim B$ if and only if $A \sim C$ and $C <: B$ for some C.
2. $A \lesssim B$ if and only if $A <: C$ and $C \sim B$ for some C.

☞ *Equivalence is broken in the polymorphic setting!*

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2. $A \lesssim B$ if and only if $A <: C$ and $C \sim B$ for some C. ✗

☞ *Equivalence is broken in the polymorphic setting!*

$$
\begin{tikzcd}
\bot \arrow[rr, dash, "\sim"] & & \boxed{(\star \to \mathsf{Int}) \to \mathsf{Int}} \\
\boxed{(\forall a.a \to \mathsf{Int}) \to \mathsf{Int}} \arrow[u, "<:"] \arrow[rr, dash, "\sim"'] & & (\forall a.\star \to \mathsf{Int}) \to \mathsf{Int} \arrow[u, "<:"]
\end{tikzcd}
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☞ *Equivalence is broken in the polymorphic setting!*

$$\begin{tikzcd}
\mathsf{Int} \to \mathsf{Int} \arrow[r, dash, "\sim"] & \boxed{\mathsf{Int} \to \star} \\
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$$\begin{tikzcd}
\bot \arrow[r, dash, "\sim"] & (((\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star) \\
(((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\forall a.a) \arrow[u, "<:"] \arrow[r, dash, "\sim"'] & \bot \arrow[u, "<:"']
\end{tikzcd}$$

Revisiting Consistent Subtyping

Consistent Subtyping vs. Subtyping

- Subtyping validates the *subsumption principle*

$$\frac{\Psi \vdash e : A \qquad A <: B}{\Psi \vdash e : B}$$

Consistent Subtyping vs. Subtyping

- Subtyping validates the *subsumption principle*, so should consistent subtyping

$$\frac{\Psi \vdash e : A \qquad A \lesssim B}{\Psi \vdash e : B}$$

Consistent Subtyping vs. Subtyping

- Subtyping validates the *subsumption principle*, so should consistent subtyping

$$\frac{\Psi \vdash e : A \qquad A \lesssim B}{\Psi \vdash e : B}$$

- Subtyping is transitive, but consistent subtyping *is not*

Observations

Observation (I)

If $A <: B$ and $B \lesssim C$, then $A \lesssim C$.

$$
\begin{tikzcd}
T_1 \arrow[r, dash, "\sim"] & C \\
B \arrow[u, "<:"] \arrow[ur, dashed, "\lesssim"] \arrow[r, dash, "\sim"'] & T_2 \arrow[u, "<:"] \\
A \arrow[u, "<:"] \arrow[uur, dashed, bend right, "\lesssim"'] &
\end{tikzcd}
$$

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\end{tikzcd}
$$

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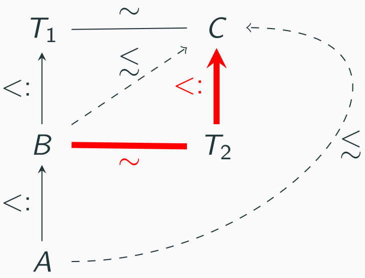

Observations

Observation (I)

If $A <: B$ and $B \lesssim C$, then $A \lesssim C$.

Observation (II)

If $C \lesssim B$ and $B <: A$, then $C \lesssim A$.

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A \arrow[u, "<:"] \arrow[uur, dashed, bend right, "\lesssim"'] &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
 & A \\
T_1 \arrow[r, dash, "\sim"] & B \arrow[u, "<:"] \\
C \arrow[u, "<:"] \arrow[r, dash, "\sim"'] \arrow[ur, dashed, "\lesssim"] \arrow[uur, dashed, bend left=60, "\lesssim"] & T_2 \arrow[u, "<:"]
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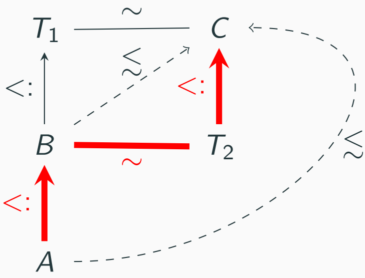
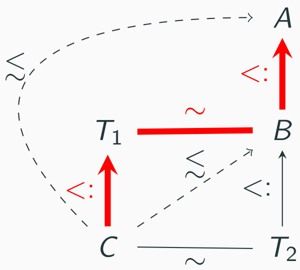
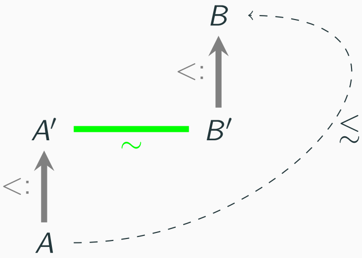




Consistent Subtyping, the Specification

Definition (Generalized Consistent Subtyping)

$\Psi \vdash A \lesssim B \stackrel{def}{=}$ $\boxed{\Psi \vdash A <: A'}$, $\boxed{A' \sim B'}$ and $\boxed{\Psi \vdash B' <: B}$ for some A' and B'.



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 & (((\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star) \\
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(((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\forall a.a) \arrow[u, "<:"] &
\end{tikzcd}$$

$$A = ((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \mathsf{Int})$$

$$B = ((\forall a.\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star)$$

Non-Determinism

Definition (Generalized Consistent Subtyping)

$\Psi \vdash A \lesssim B \overset{def}{=} \Psi \vdash A <: A'$, $A' \sim B'$ and $\Psi \vdash B' <: B$ for some A' and B'.

Two sources of non-determinism:

1. Two intermediate types A' and B'

Non-Determinism

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Two sources of non-determinism:

1. Two intermediate types A' and B'
2. Guessing monotypes $\dfrac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] <: B}{\Psi \vdash \forall a.\, A <: B}$

Non-Determinism

Definition (Generalized Consistent Subtyping)

$\Psi \vdash A \lesssim B \overset{def}{=} \boxed{\Psi \vdash A <: A'}$, $\boxed{A' \sim B'}$ and $\boxed{\Psi \vdash B' <: B}$ for some A' and B'.

Two sources of non-determinism:

1. Two intermediate types A' and B'

☞ *We can derive a syntax-directed inductive definition without resorting to subtyping or consistency at all!*

Consistent Subtyping Without Existentials

Notice $\Psi \vdash \star \lesssim A$ always holds ($\star <: \star \sim A <: A$), and vise versa ($\Psi \vdash A \lesssim \star$)

Consistent Subtyping Without Existentials: First Step

1. Replace $<:$ with $\lesssim$

$\boxed{\Psi \vdash A <: B}$ *(Subtyping)*

$$\frac{a \in \Psi}{\Psi \vdash a <: a} \qquad \frac{}{\Psi \vdash \mathsf{Int} <: \mathsf{Int}} \qquad \frac{\Psi \vdash B_1 <: A_1 \qquad \Psi \vdash A_2 <: B_2}{\Psi \vdash A_1 \to A_2 <: B_1 \to B_2}$$

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$$\boxed{\frac{}{\Psi \vdash \star <: \star}}$$

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$\boxed{\Psi \vdash A \lesssim B}$ *(Consistent Subtyping, not yet)*

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$$\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] \lesssim B}{\Psi \vdash \forall a.\, A \lesssim B} \qquad \frac{\Psi, a \vdash A \lesssim B}{\Psi \vdash A \lesssim \forall a.\, B}$$

$$\frac{}{\Psi \vdash \star \lesssim \star}$$

Consistent Subtyping Without Existentials: Second Step

1. Replace $<:$ with $\lesssim$
2. Replace $\Psi \vdash \star \lesssim \star$ with $\Psi \vdash \star \lesssim A$ and $\Psi \vdash A \lesssim \star$

$\boxed{\Psi \vdash A \lesssim B}$ *(Consistent Subtyping, not yet)*

$$\frac{a \in \Psi}{\Psi \vdash a \lesssim a} \qquad \frac{}{\Psi \vdash \mathsf{Int} \lesssim \mathsf{Int}} \qquad \frac{\Psi \vdash B_1 \lesssim A_1 \qquad \Psi \vdash A_2 \lesssim B_2}{\Psi \vdash A_1 \to A_2 \lesssim B_1 \to B_2}$$

$$\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] \lesssim B}{\Psi \vdash \forall a.\, A \lesssim B} \qquad \frac{\Psi, a \vdash A \lesssim B}{\Psi \vdash A \lesssim \forall a.\, B}$$

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Consistent Subtyping Without Existentials: Second Step

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$\boxed{\Psi \vdash A \lesssim B}$ *(Consistent Subtyping)*

$$\frac{a \in \Psi}{\Psi \vdash a \lesssim a} \qquad \frac{}{\Psi \vdash \mathsf{Int} \lesssim \mathsf{Int}} \qquad \frac{\Psi \vdash B_1 \lesssim A_1 \qquad \Psi \vdash A_2 \lesssim B_2}{\Psi \vdash A_1 \to A_2 \lesssim B_1 \to B_2}$$

$$\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] \lesssim B}{\Psi \vdash \forall a.\, A \lesssim B} \qquad \frac{\Psi, a \vdash A \lesssim B}{\Psi \vdash A \lesssim \forall a.\, B}$$

$$\frac{}{\Psi \vdash \star \lesssim A} \qquad \frac{}{\Psi \vdash A \lesssim \star}$$

Definition Meets Specification

Theorem

$\Psi \vdash A \lesssim B$ *iff* $\Psi \vdash A <: A'$, $A' \sim B'$ *and* $\Psi \vdash B' <: B$ *for some* A' *and* B'.

Declarative Type System

Type System

$\boxed{\Psi \vdash e : A}$ *(Typing, selected rules)*

$$\frac{\Psi, a \vdash e : A}{\Psi \vdash e : \forall a.\, A}\ \textsc{u-gen} \qquad \frac{\Psi, x : A \vdash e : B}{\Psi \vdash \lambda x : A.\, e : A \to B}\ \textsc{u-lamann}$$

$$\frac{\Psi, x : \tau \vdash e : B}{\Psi \vdash \lambda x.\, e : \tau \to B}\ \textsc{u-lam} \qquad \frac{\begin{array}{cc}\Psi \vdash e_1 : A & \Psi \vdash A \rhd A_1 \to A_2 \\ \Psi \vdash e_2 : A_3 & \Psi \vdash A_3 \lesssim A_1\end{array}}{\Psi \vdash e_1\ e_2 : A_2}\ \textsc{u-app}$$

Type System

$$\frac{\Psi \vdash e_1 : A \qquad \Psi \vdash A \rhd A_1 \to A_2 \qquad \Psi \vdash e_2 : A_3 \qquad \Psi \vdash A_3 \lesssim A_1}{\Psi \vdash e_1\ e_2 : A_2}\ \textsc{U-App}$$

$\boxed{\Psi \vdash A \rhd A_1 \to A_2}$ *(Matching)*

$$\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] \rhd A_1 \to A_2}{\Psi \vdash \forall a.\, A \rhd A_1 \to A_2}\ \textsc{M-Forall}$$

$$\frac{}{\Psi \vdash A_1 \to A_2 \rhd A_1 \to A_2}\ \textsc{M-Arr} \qquad \frac{}{\Psi \vdash \star \rhd \star \to \star}\ \textsc{M-Unknown}$$

Dynamic Semantics

- Type-directed translation into an intermediate language with runtime casts ($\Psi \vdash e : A \rightsquigarrow s$)
- We translate to the Polymorphic Blame Calculus (PBC) [Ahmed et al., 2011]

$$\text{PBC terms}^2 \quad s ::= x \mid \mathsf{n} \mid \lambda x : A.\, s \mid \Lambda a.\, s \mid s_1\, s_2 \mid \langle A \hookrightarrow B \rangle s$$

[2] Only a subst of PBC terms are used

Correctness Criteria

- **Conservative extension:** for all static Ψ, e, and A,
  - if $\Psi \vdash^{OL} e : A$, then there exists B, such that $\Psi \vdash e : B$, and $\Psi \vdash B <: A$.
  - if $\Psi \vdash e : A$, then $\Psi \vdash^{OL} e : A$
- **Monotonicity w.r.t. precision:** for all Ψ, e, e', A, if $\Psi \vdash e : A$, and $e' \sqsubseteq e$, then $\Psi \vdash e' : B$, and $B \sqsubseteq A$ for some B.
- **Type Preservation of cast insertion:** for all Ψ, e, A, if $\Psi \vdash e : A$, then $\Psi \vdash e : A \rightsquigarrow s$, and $\Psi \vdash^{B} s : A$ for some s.
- **Monotonicity of cast insertion:** for all $\Psi, e_1, e_2, s_1, s_2, A$, if $\Psi \vdash e_1 : A \rightsquigarrow s_1$, and $\Psi \vdash e_2 : A \rightsquigarrow s_2$, and $e_1 \sqsubseteq e_2$, then $\Psi \mid \Psi \vdash s_1 \sqsubseteq^{B} s_2$.

Correctness Criteria

- **Conservative extension:** for all static Ψ, e, and A,
  - if $\Psi \vdash^{OL} e : A$, then there exists B, such that $\Psi \vdash e : B$, and $\Psi \vdash B <: A$.
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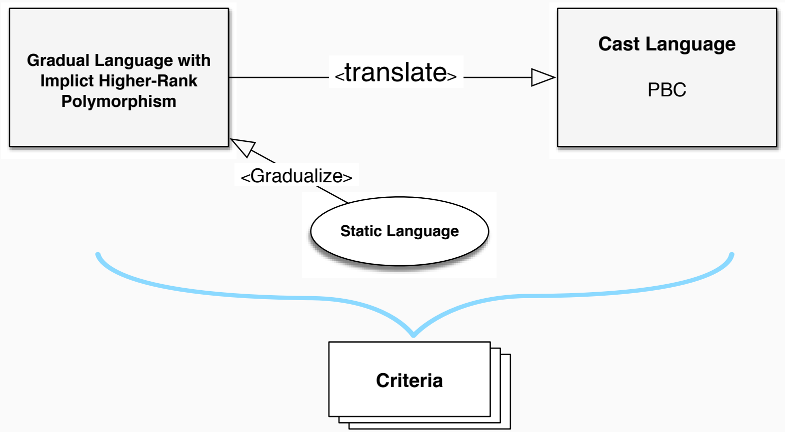
- **Monotonicity of cast insertion:** for all $\Psi, e_1, e_2, s_1, s_2, A$, if $\Psi \vdash e_1 : A \rightsquigarrow s_1$, and $\Psi \vdash e_2 : A \rightsquigarrow s_2$, and $e_1 \sqsubseteq e_2$, then $\Psi \mid \Psi \vdash s_1 \sqsubseteq^{B} s_2$.

☞ *Proved in Coq!*

Recap


Gradual Language with Implict Higher-Rank Polymorphism
<translate>
Cast Language
PBC
<Gradualize>
Static Language
Criteria

More in the Paper

- A bidirectional account of the algorithmic type system (inspired by [Dunfield and Krishnaswami, 2013])
- Extension to top types
- Discussion and comparison with other approaches (AGT [Garcia et al., 2016], Directed Consistency [Jafery and Dunfield, 2017])
- Discussion of dynamic guarantee

Future Work

- Fix the issue with dynamic guarantee (partially)
- More features: mutable state, fancy types, etc.

References

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Consistent Subtyping for All

Ningning Xie **Xuan Bi** Bruno C. d. S. Oliveira

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The University of Hong Kong
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Backup Slides

Dynamic Guarantee

- Changes to the annotations of a gradually typed program should not change the dynamic behaviour of the program.
- The declarative system breaks it...

$$(\lambda f : \forall a.\, a \to \mathsf{Int}.\, \lambda x : \mathsf{Int}.\, f\ x)\,(\lambda x.\, 1)\, 3 \Downarrow 3$$

$$(\lambda f : \forall a.\, a \to \mathsf{Int}.\, \lambda x : \star.\, f\ x)\,(\lambda x.\, 1)\, 3 \Downarrow ?$$

- A common problem in gradual type inference, see [Garcia and Cimini 2015]. Static and gradual type parameters may help.
- A more sophisticated term precision is needed in PBC. [Igarashi et al. 2017]